CECW-ED Engineer Manual 1110-2-2104	Department of the Army U.S. Army Corps of Engineers Washington, DC 20314-1000	EM 1110-2-2104 30 June 1992
	Engineering and Design STRENGTH DESIGN FOR REINFORCED CONCRETE HYDRAULIC STRUCTURES	
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EM 1110-2-2104
30 June 1992

US Army Corps
of Engineers

ENGINEERING AND DESIGN

Strength Design for Reinforced-Concrete Hydraulic Structures

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ENGINEER MANUAL

DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

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CECW-ED

Engineer Manual
No. 1110-2-2104

30 June 1992

Engineering and Design
STRENGTH DESIGN FOR
REINFORCED-CONCRETE HYDRAULIC STRUCTURES

1. Purpose. This manual provides guidance for designing reinforced concrete hydraulic structures by the strength-design method. Plain concrete and prestressed concrete are not covered in this manual.

2. Applicability. This manual applies to all HQUSACE/OCE elements, major subordinate commands, districts, laboratories, and field operating activities having civil works responsibilities.

FOR THE COMMANDER:

Milton Hunter
MILTON HUNTER
Colonel, Corps of Engineers
Chief of Staff

This manual supersedes ETL 1110-2-312, Strength Design Criteria for Reinforced Concrete Hydraulic Structures, dated 10 March 1988 and EM 1110-2-2103, Details of Reinforcement-Hydraulic Structures, dated 21 May 1971.

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CHAPTER 1

INTRODUCTION

1-1. Purpose

This manual provides guidance for designing reinforced-concrete hydraulic structures by the strength-design method.

1-2. Applicability

This manual applies to all HQUSACE/OCE elements, major subordinate commands, districts, laboratories, and field operating activities having civil works responsibilities.

1-3. References

a. EM 1110-1-2101, Working Stresses for Structural Design.

b. EM 1110-2-2902, Conduits, Culverts, and Pipes.

c. CW-03210, Civil Works Construction Guide Specification for Steel Bars, Welded Wire Fabric, and Accessories for Concrete Reinforcement.

d. American Concrete Institute, "Building Code Requirements and Commentary for Reinforced Concrete," ACI 318, Box 19150, Redford Station, Detroit, MI 48219.

e. American Concrete Institute, "Environmental Engineering Concrete Structures," ACI 350R, Box 19150, Redford Station, Detroit, MI 48219.

f. American Society for Testing and Materials, "Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement," ASTM A 615-89, 1916 Race St., Philadelphia, PA 19103.

g. American Welding Society, "Structural Welding Code-Reinforcing Steel," AWS D1.4-790, 550 NW Le Jeune Rd., P.O. Box 351040, Miami, FL 33135.

h. Liu, Tony C. 1980 (Jul). "Strength Design of Reinforced Concrete Hydraulic Structures, Report 1: Preliminary Strength Design Criteria," Technical Report SL-80-4, US Army Engineer Waterways Experiment Station, 3909 Halls Ferry Road, Vicksburg, MS 39180.

i. Liu, Tony C., and Gleason, Scott. 1981 (Sep). "Strength Design of Reinforced Concrete Hydraulic Structures, Report 2: Design Aids for Use in the Design and Analysis of Reinforced Concrete Hydraulic Structural Members Subjected to Combined Flexural and Axial Loads," Technical Report SL-80-4, US Army Engineer Waterways Experiment Station, 3909 Halls Ferry Road, Vicksburg, MS 39180.

j. Liu, Tony C. 1981 (Sep). "Strength Design of Reinforced Concrete Hydraulic Structures, Report 3: T-Wall Design," Technical Report SL-80-4, US Army Engineer Waterways Experiment Station, 3909 Halls Ferry Road, Vicksburg, MS 39180.

1-4. Background

a. A reinforced concrete hydraulic structure is one that will be subjected to one or more of the following: submergence, wave action, spray, chemically contaminated atmosphere, and severe climatic conditions. Typical hydraulic structures are stilling-basin slabs and walls, concrete-lined channels, portions of powerhouses, spillway piers, spray walls and training walls, floodwalls, intake and outlet structures below maximum high water and wave action, lock walls, guide and guard walls, and retaining walls subject to contact with water.

b. In general, existing reinforced-concrete hydraulic structures designed by the Corps, using the working stress method of EM 1110-1-2101, have held up extremely well. The Corps began using strength design methods in 1981 (Liu 1980, 1981 and Liu and Gleason 1981) to stay in step with industry, universities, and other engineering organizations. ETL 1110-2-265, "Strength Design Criteria for Reinforced Concrete Hydraulic Structures," dated 15 September 1981, was the first document providing guidance issued by the Corps concerning the use of strength design methods for hydraulic structures. The labor-intensive requirements of this ETL regarding the application of multiple load factors, as well as the fact that some load-factor combination conditions resulted in a less conservative design than if working stress methods were used, resulted in the development of ETL 1110-2-312, "Strength Design Criteria for Reinforced Concrete Hydraulic Structures," dated 10 March 1988.

c. The revised load factors in ETL 1110-2-312 were intended to ensure that the resulting design was as conservative as if working stress methods were used. Also, the single load factor concept was introduced. The guidance in this ETL differed from ACI 318 Building Code Requirements and Commentary for Reinforced Concrete primarily in the load factors, the concrete stress-strain relationship, and the yield strength of Grade 60 reinforcement. ETL 1110-2-312 guidance was intended to result in designs equivalent to those resulting when working stress methods were used.

d. Earlier Corps strength design methods deviated from ACI guidance because ACI 318 includes no provisions for the serviceability needs of hydraulic structures. Strength and stability are required, but serviceability in terms of deflections, cracking, and durability demand equal consideration. The importance of the Corps' hydraulic structures has caused the Corps to move cautiously, but deliberately, toward exclusive use of strength design methods.

e. This manual modifies and expands the guidance in ETL 1110-2-312 with an approach similar to that of ACI 350R-89. The concrete stress-strain relationship and the yield strength of Grade 60 reinforcement given in ACI 318 are adopted. Also, the load factors bear a closer resemblance to ACI 318 and

are modified by a hydraulic factor, H_f, to account for the serviceability needs of hydraulic structures.

f. As in ETL 1110-2-312, this manual allows the use of a single load factor for both dead and live loads. In addition, the single load factor method is required when the loads on the structural component include reactions from a soil-structure stability analysis.

1-5. General Requirements

Reinforced-concrete hydraulic structures should be designed with the strength design method in accordance with the current ACI 318, except as hereinafter specified. The notations used are the same as those used in the ACI 318 Code and Commentary, except those defined herein.

1-6. Scope

a. This manual is written in sufficient detail to not only provide the designer with design procedures, but to also provide examples of their application. Also, derivations of the combined flexural and axial load equations are given to increase the designer's confidence and understanding.

b. General detailing requirements are presented in Chapter 2. Chapter 3 presents strength and serviceability requirements, including load factors and limits on flexural reinforcement. Design equations for members subjected to flexural and/or axial loads (including biaxial bending) are given in Chapter 4. Chapter 5 presents guidance for design for shear, including provisions for curved members and special straight members. The appendices include notation, equation derivations, and examples. The examples demonstrate: load-factor application, design of members subjected to combined flexural and axial loads, design for shear, development of an interaction diagram, and design of members subjected to biaxial bending.

1-7. Computer Programs

Copies of computer programs, with documentation, for the analysis and design of reinforced-concrete hydraulic structures are available and may be obtained from the Engineering Computer Programs Library, US Army Engineer Waterways Experiment Station, 3909 Halls Ferry Road, Vicksburg, Mississippi 39180-6199. For design to account for combined flexural and axial loads, any procedure that is consistent with ACI 318 guidance is acceptable, as long as the load factor and reinforcement percentage guidance given in this manual is followed.

1-8. Recission

Corps library computer program CSTR (X0066), based on ETL 1110-2-312, is replaced by computer program CASTR (X0067). Program CASTR is based on this new engineer manual.

CHAPTER 2

DETAILS OF REINFORCEMENT

2-1. General

This chapter presents guidance for furnishing and placing steel reinforcement in various concrete members of hydraulic structures.

2-2. Quality

The type and grade of reinforcing steel should be limited to ASTM A 615 (Billet Steel), Grade 60. Grade 40 reinforcement should be avoided since its availability is limited and designs based on Grade 40 reinforcement, utilizing the procedures contained herein, would be overly conservative. Reinforcement of other grades and types permitted by ACI 318 may be permitted for special applications subject to the approval of higher authority.

2-3. Anchorage, Bar Development, and Splices

The anchorage, bar development, and splice requirements should conform to ACI 318 and to the requirements presented below. Since the development length is dependent on a number of factors such as concrete strength and bar position, function, size, type, spacing, and cover, the designer must indicate the length of embedment required for bar development on the contract drawings. For similar reasons, the drawings should show the splice lengths and special requirements such as staggering of splices, etc. The construction specifications should be carefully edited to assure that they agree with reinforcement details shown on the drawings.

2-4. Hooks and Bends

Hooks and bends should be in accordance with ACI 318.

2-5. Bar Spacing

a. Minimum. The clear distance between parallel bars should not be less than 1-1/2 times the nominal diameter of the bars nor less than 1-1/2 times the maximum size of coarse aggregate. No. 14 and No. 18 bars should not be spaced closer than 6 and 8 inches, respectively, center to center. When parallel reinforcement is placed in two or more layers, the clear distance between layers should not be less than 6 inches. In horizontal layers, the bars in the upper layers should be placed directly over the bars in the lower layers. In vertical layers, a similar orientation should be used. In construction of massive reinforced concrete structures, bars in a layer should be spaced 12 inches center-to-center wherever possible to facilitate construction.

b. Maximum. The maximum center-to-center spacing of both primary and secondary reinforcement should not exceed 18 inches.

2-6. Concrete Protection for Reinforcement

The minimum cover for reinforcement should conform to the dimensions shown below for the various concrete sections. The dimensions indicate the clear distance from the edge of the reinforcement to the surface of the concrete.

CONCRETE SECTION	MINIMUM CLEAR COVER OF REINFORCEMENT, INCHES
Unformed surfaces in contact with foundation	4
Formed or screeded surfaces subject to cavitation or abrasion erosion, such as baffle blocks and stilling basin slabs	6
Formed and screeded surfaces such as stilling basin walls, chute spillway slabs, and channel lining slabs on grade:	
Equal to or greater than 24 inches in thickness	4
Greater than 12 inches and less than 24 inches in thickness	3
Equal to or less than 12 inches in thickness will be in accordance with ACI Code 318.	

NOTE. In no case shall the cover be less than:
1.5 times the nominal maximum size of aggregate,
or
2.5 times the maximum diameter of reinforcement.

2-7. Splicing

a. General. Bars shall be spliced only as required and splices shall be indicated on contract drawings. Splices at points of maximum tensile stress should be avoided. Where such splices must be made they should be staggered. Splices may be made by lapping of bars or butt splicing.

b. Lapped Splices. Bars larger than No. 11 shall not be lap-spliced. Tension splices should be staggered longitudinally so that no more than half of the bars are lap-spliced at any section within the required lap length. If staggering of splices is impractical, applicable provisions of ACI 318 should be followed.

c. Butt Splices

(1) General. Bars larger than No. 11 shall be butt-spliced. Bars No. 11 or smaller should not be butt-spliced unless clearly justified by design details or economics. Due to the high costs associated with butt splicing of bars larger than No. 11, especially No. 18 bars, careful

consideration should be given to alternative designs utilizing smaller bars. Butt splices should be made by either the thermit welding process or an approved mechanical butt-splicing method in accordance with the provisions contained in the following paragraphs. Normally, arc-welded splices should not be permitted due to the inherent uncertainties associated with welding reinforcement. However, if arc welding is necessary, it should be done in accordance with AWS D1.4, Structural Welding Code-Reinforcing Steel. Butt splices should develop in tension at least 125 percent of the specified yield strength, f_y, of the bar. Tension splices should be staggered longitudinally at least 5 feet for bars larger than No. 11 and a distance equal to the required lap length for No. 11 bars or smaller so that no more than half of the bars are spliced at any section. Tension splices of bars smaller than No. 14 should be staggered longitudinally a distance equal to the required lap length. Bars Nos. 14 and 18 shall be staggered longitudinally, a minimum of 5 feet so that no more than half of the bars are spliced at any one section.

(2) Thermit Welding. Thermit welding should be restricted to bars conforming to ASTM A 615 (billet steel) with a sulfur content not exceeding 0.05 percent based on ladle analysis. The thermit welding process should be in accordance with the provisions of Guide Specification CW-03210.

(3) Mechanical Butt Splicing. Mechanical butt splicing shall be made by an approved exothermic, threaded coupling, swaged sleeve, or other positive connecting type in accordance with the provisions of Guide Specification CW-03210. The designer should be aware of the potential for slippage in mechanical splices and insist that the testing provisions contained in this guide specification be included in the contract documents and utilized in the construction work.

2-8. Temperature and Shrinkage Reinforcement

a. In the design of structural members for temperature and shrinkage stresses, the area of reinforcement should be 0.0028 times the gross cross-sectional area, half in each face, with a maximum area equivalent to No. 9 bars at 12 inches in each face. Generally, temperature and shrinkage reinforcement for thin sections will be no less than No. 4 bars at 12 inches in each face.

b. Experience and/or analyses may indicate the need for an amount of reinforcement greater than indicated in paragraph 2-8a if the reinforcement is to be used for distribution of stresses as well as for temperature and shrinkage.

c. In general, additional reinforcement for temperature and shrinkage will not be needed in the direction and plane of the primary tensile reinforcement when restraint is accounted for in the analyses. However, the primary reinforcement should not be less than that required for shrinkage and temperature as determined above.

CHAPTER 3

STRENGTH AND SERVICEABILITY REQUIREMENTS

3-1. General

a. All reinforced-concrete hydraulic structures must satisfy both strength and serviceability requirements. In the strength design method, this is accomplished by multiplying the service loads by appropriate load factors and by a hydraulic factor, H_f. The hydraulic factor is applied to the overall load factor equations for obtaining the required nominal strength. The hydraulic factor is used to improve crack control in hydraulic structures by increasing reinforcement requirements, thereby reducing steel stresses at service load levels.

b. Two methods are available for determining the factored moments, shears, and thrusts for designing hydraulic structures by the strength design method. They are the single load factor method and a method based on the ACI 318 Building Code. Both methods are described herein.

c. In addition to strength and serviceability requirements, many hydraulic structures must also satisfy stability requirements under various loading and foundation conditions. The loads from stability analyses that are used to design structural components by the strength design method must be obtained as prescribed herein to assure correctness of application.

3-2. Stability Analysis

a. The stability analysis of structures, such as retaining walls, must be performed using unfactored loads. The unfactored loads and the resulting reactions are then used to determine the unfactored moments, shears, and thrusts at critical sections of the structure. The unfactored moments, shears, and thrusts are then multiplied by the appropriate load factors, and the hydraulic factor when appropriate, to determine the required strengths which, in turn, are used to establish the required section properties.

b. The single load factor method must be used when the loads on the structural component being analyzed include reactions from a soil-structure interaction stability analysis, such as footings for walls. For simplicity and ease of application, this method should generally be used for all elements of such structures. The load factor method based on the ACI 318 Building Code may be used for some elements of the structure, but must be used with caution to assure that the load combinations do not produce unconservative results.

3-3. Required Strength

Reinforced-concrete hydraulic structures and hydraulic structural members shall be designed to have a required strength, U_h, to resist dead and live loads in accordance with the following provisions. The hydraulic factor is to be applied in the determination of required nominal strength for all combinations of axial load and moment, as well as for diagonal tension

(shear). However, the required design strength for reinforcement in diagonal tension (shear) should be calculated by applying a hydraulic factor of 1.3 to the excess shear. Excess shear is defined as the difference between the factored shear force at the section, V_u, and the shear strength provided by the concrete, ϕV_c. Thus $\phi V_s \geq 1.3\ (V_u - \phi V_c)$, where ϕV_s is the design capacity of the shear reinforcement.

a. Single load factor method. In the single load factor method, both the dead and live loads are multiplied by the same load factor.

$$U = 1.7(D + L) \tag{3-1}$$

where

D = internal forces and moments from dead loads of the concrete members only

L = internal forces and moments from live loads (loads other than the dead load of the concrete member)

For hydraulic structures, the basic load factor is then multiplied by a hydraulic factor, H_f.

$$U_h = H_f(U) \tag{3-2}$$

where H_f = 1.3 for hydraulic structures, except for members in direct tension. For members in direct tension, H_f = 1.65. Other values may be used subject to consultation with and approval by CECW-ED.

Therefore, the required strength U_h to resist dead and live loads shall be at least equal to

$$U_h = 1.7H_f(D + L) \tag{3-3}$$

An exception to the above occurs when resistance to the effects of wind or other forces that constitute short duration loads with low probability of occurrence are included in the design. For that case, the following loading combination should be used:

$$U_h = 0.75\left[1.7H_f(D + L)\right] \tag{3-4}$$

b. Modified ACI 318 Building Code Method. The load factors prescribed in ACI 318 may be applied directly to hydraulic structures with two modifications. The load factor for lateral fluid pressure, F, should be taken as 1.7. The factored load combinations for total factored design load, U, as

prescribed in ACI 318 shall be increased by the hydraulic factor $H_f = 1.3$, except for members in direct tension. For members in direct tension, $H_f = 1.65$.

The equations for required strength can be expressed as

$$U_h = 1.3U \tag{3-5}$$

except for members in direct tension where

$$U_h = 1.65(U) \tag{3-6}$$

For certain hydraulic structures such as U-frame locks and U-frame channels, the live load can have a relieving effect on the factored load combination used to determine the total factored load effects (shears, thrusts, and moments). In this case, the combination of factored dead and live loads with a live load factor of unity should be investigated and reported in the design documents.

c. Earthquake effects. If a resistance to specified earthquake loads or forces, E, are included, the following combinations shall apply.

(1) Unusual

(a) Nonsite-specific ground motion design earthquake (OBE)

$$U_h = 1.7(D + L) + 1.9E \tag{3-7}$$

(b) Site-specific ground motion for design earthquake with time-history and response spectrim analysis (OBE)

$$U_h = 1.4(D+L) + 1.5E \tag{3-8}$$

(2) Extreme

(a) Nonsite-specific ground motion (MCE)

$$U_h = 1.1(D + L) + 1.25E \tag{3-9}$$

(b) Site-specific ground motion (MCE)

$$U_h = 1.0(D + L + E) \tag{3-10}$$

d. Nonhydraulic structures. Reinforced concrete structures and structural members that are not classified as hydraulic shall be designed with the above guidance, except that the hydraulic factor shall not be used.

3-4. Design Strength of Reinforcement

a. Design should normally be based on 60,000 psi, the yield strength of ASTM Grade 60 reinforcement. Other grades may be used, subject to the provisions of paragraphs 2-2 and 3-4.b. The yield strength used in the design shall be indicated on the drawings.

b. Reinforcement with a yield strength in excess of 60,000 psi shall not be used unless a detailed investigation of ductility and serviceability requirements is conducted in consultation with and approved by CECW-ED.

3-5. Maximum Tension Reinforcement

a. For singly reinforced flexural members, and for members subject to combined flexure and compressive axial load when the axial load strength ϕPn is less than the smaller of $0.10 f'_c A_g$ or ϕPb, the ratio of tension reinforcement ρ provided shall conform to the following.

(1) Recommended limit = 0.25 ρ_b.

(2) Maximum permitted upper limit not requiring special study or investigation = 0.375 ρ_b. Values above 0.375 ρ_b will require consideration of serviceability, constructibility, and economy.

(3) Maximum permitted upper limit when excessive deflections are not predicted when using the method specified in ACI 318 or other methods that predict deformations in substantial agreement with the results of comprehensive tests = 0.50 ρ_b.

(4) Reinforcement ratios above 0.5 ρ_b shall only be permitted if a detailed investigation of serviceability requirements, including computation of deflections, is conducted in consultation with and approved by CECW-ED. Under no circumstance shall the reinforcement ratio exceed 0.75 ρ_b.

b. Use of compression reinforcement shall be in accordance with provisions of ACI 318.

3-6. Control of Deflections and Cracking

a. Cracking and deflections due to service loads need not be investigated if the limits on the design strength and ratio of the reinforcement specified in paragraphs 3-4.a and 3-5.a(3) are not exceeded.

b. For design strengths and ratios of reinforcement exceeding the limits specified in paragraphs 3-4.a and 3-5.a(3), extensive investigations of deformations and cracking due to service loads should be made in consultation with CECW-ED. These investigations should include laboratory tests of materials and models, analytical studies, special construction procedures, possible measures for crack control, etc. Deflections and crack widths should be limited to levels which will not adversely affect the operation, maintenance, performance, or appearance of that particular structure.

3-7. Minimum Thickness of Walls

Walls with height greater than 10 feet shall be a minimum of 12 inches thick and shall contain reinforcement in both faces.

CHAPTER 4

FLEXURAL AND AXIAL LOADS

4-1. Design Assumptions and General Requirements

a. The assumed maximum usable strain ε_c at the extreme concrete compression fiber should be equal to 0.003 in accordance with ACI 318.

b. Balanced conditions for hydraulic structures exist at a cross section when the tension reinforcement ρ_b reaches the strain corresponding to its specified yield strength f_y just as the concrete in compression reaches its design strain ε_c.

c. Concrete stress of $0.85f'_c$ should be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.

d. Factor β_1 should be taken as specified in ACI 318.

e. The eccentricity ratio e'/d should be defined as

$$\frac{e'}{d} = \frac{M_u/P_u + d - h/2}{d} \qquad (4\text{-}11)^*$$

where e' = eccentricity of axial load measured from the centroid of the tension reinforcement

4-2. Flexural and Compressive Capacity - Tension Reinforcement Only

a. The design axial load strength ϕP_n at the centroid of compression members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi\left[0.85f'_c(A_g - \rho bd) + f_y\rho bd\right] \qquad (4\text{-}12)$$

b. The strength of a cross section is controlled by compression if the load has an eccentricity ratio e'/d no greater than that given by Equation 4-3 and by tension if e'/d exceeds this value.

---

* P_u is considered positive for compression and negative for tension.

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2}{2k_b - \dfrac{\rho f_y}{0.425 f_c'}} \tag{4-13}$$

where

$$k_b = \frac{\beta_1 E_s \varepsilon_c}{E_s \varepsilon_c + f_y} \tag{4-14}$$

c. Sections controlled by tension should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u - \rho f_y\right) bd \tag{4-15}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u - \rho f_y\right) \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-16}$$

where k_u should be determined from the following equation:

$$k_u = \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c'}\right)\frac{e'}{d}} - \left(\frac{e'}{d} - 1\right) \tag{4-17}$$

d. Sections controlled by compression should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u - \rho f_s\right) bd \tag{4-18}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u - \rho f_s\right) \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-19}$$

where

$$f_s = \frac{E_s \varepsilon_c (\beta_1 - k_u)}{k_u} \geq -f_y \tag{4-20}$$

and k_u should be determined from the following equation by direct or iterative method:

$$k_u^3 + 2\left(\frac{e'}{d} - 1\right)k_u^2 + \left(\frac{E_s \varepsilon_c \rho e'}{0.425 f_c' d}\right)k_u - \frac{\beta_1 E_s \varepsilon_c \rho e'}{0.425 f_c' d} = 0 \tag{4-21}$$

e. The balanced load and moment can be computed using either Equations 4-5 and 4-6 or Equations 4-8 and 4-9 with $k_u = k_b$ and $\frac{e'}{d} = \frac{e_b'}{d}$. The values of e_b'/d and k_b are given by Equations 4-3 and 4-4, respectively.

4-3. Flexural and Compressive Capacity - Tension and Compression Reinforcement

a. The design axial load strength ϕP_n of compression members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi\left\{0.85 f_c'\left[A_g - (\rho + \rho')bd\right] + f_y(\rho + \rho')bd\right\} \tag{4-22}$$

b. The strength of a cross section is controlled by compression if the load has an eccentricity ratio e'/d no greater than that given by Equation 4-13 and by tension if e'/d exceeds this value.

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2 + \dfrac{\rho' f_s'\left(1 - \dfrac{d'}{d}\right)}{0.425 f_c'}}{2k_b - \dfrac{\rho f_y}{0.425 f_c'} + \dfrac{\rho' f_s'}{0.425 f_c'}} \tag{4-23}$$

The value k_b is given in Equation 4-4 and f_s' is given in Equation 4-16 with $k_u = k_b$.

c. Sections controlled by tension should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_y\right) bd \tag{4-24}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_y\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-25}$$

where

$$f_s' = \frac{\left(k_u - \beta_1 \frac{d'}{d}\right)}{\left(\beta_1 - k_u\right)} E_s \varepsilon_y \leq f_y \tag{4-26}$$

and k_u should be determined from the following equation by direct or iterative methods:

$$\begin{aligned} & k_u^3 + \left[2\left(\frac{e'}{d} - 1\right) - \beta_1\right] k_u^2 - \left\{\frac{f_y}{0.425 f_c}\left[\rho'\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right]\right. \\ & \left. + 2\,\beta_1\left(\frac{e'}{d} - 1\right)\right\} k_u + \frac{f_y \beta_1}{0.425 f_c}\left[\frac{\rho' d'}{d}\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right] \\ & = 0 \end{aligned} \tag{4-27}$$

d. Sections controlled by compression should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_s\right) bd \tag{4-28}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_s\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-29}$$

where

$$f_s = \frac{E_s \varepsilon_c (\beta_1 - k_u)}{k_u} \geq -f_y \tag{4-30}$$

and

$$f_s' = \frac{E_s \varepsilon_c \left[k_u - \beta_1\left(\frac{d'}{d}\right)\right]}{k_u} \leq f_y \tag{4-31}$$

and k_u should be determined from the following equation by direct or iterative methods:

$$\begin{aligned} k_u^3 &+ 2\left(\frac{e'}{d} - 1\right) k_u^2 + \frac{E_s \varepsilon_c}{0.425 f_c'}\left[(\rho + \rho')\left(\frac{e'}{d}\right)\right. \\ &\left. - \rho'\left(1 - \frac{d'}{d}\right)\right] k_u - \frac{\beta_1 E_s \varepsilon_c}{0.425 f_c'}\left[\rho'\left(\frac{d'}{d}\right)\left(\frac{e'}{d}\right.\right. \\ &\left.\left. + \frac{d'}{d} - 1\right) + \rho\left(\frac{e'}{d}\right)\right] = 0 \end{aligned} \tag{4-32}$$

Design for flexure utilizing compression reinforcement is discouraged. However, if compression reinforcement is used in members controlled by compression, lateral reinforcement shall be provided in accordance with the ACI Building Code.

e. The balanced load and moment should be computed using Equations 4-14, 4-15, 4-16, and 4-17 with $k_u = k_b$ and $\frac{e'}{d} = \frac{e_b'}{d}$. The values of e_b'/d and k_b are given by Equations 4-13 and 4-4, respectively.

4-4. Flexural and Tensile Capacity

a. The design axial strength ϕP_n of tensile members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi(\rho + \rho')f_y bd \tag{4-33}$$

b. Tensile reinforcement should be provided in both faces of the member if the load has an eccentricity ratio e'/d in the following range:

$$\left(1 - \frac{h}{2d}\right) \geq \frac{e'}{d} \geq 0$$

The section should be designed so

$$\phi P_n = \phi(\rho f_y + \rho' f_s')bd \tag{4-24}$$

and

$$\phi M_n = \phi(\rho f_y + \rho' f_s')\left[\left(1 - \frac{h}{2d}\right) - \frac{e'}{d}\right]bd^2 \tag{4-25}$$

with

$$f_s' = f_y \frac{\left(k_u + \frac{d'}{d}\right)}{(k_u + 1)} \geq -f_y \tag{4-26}$$

and k_u should be determined from the following equation:

$$k_u = \frac{\rho' \frac{d'}{d}\left(1 - \frac{d'}{d} - \frac{e'}{d}\right) - \rho \frac{e'}{d}}{\rho \frac{e'}{d} - \rho'\left(1 - \frac{d'}{d} - \frac{e'}{d}\right)} \tag{4-27}$$

c. Sections subjected to a tensile load with an eccentricity ratio $e'/d < 0$ should be designed using Equations 4-5 and 4-6. The value of k_u is

$$k_u = -\left(\frac{e'}{d} - 1\right) - \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c}\right)\frac{e'}{d}} \tag{4-28}$$

d. Sections subject to a tensile load with an eccentricity ratio $e'/d < 0$ should be designed using Equations 4-14, 4-15, 4-16, and 4-17 if $A_s' > 0$ and $c > d'$.

4-5. Biaxial Bending and Axial Load

a. Provisions of paragraph 4-5 shall apply to reinforced concrete members subjected to biaxial bending.

b. For a given nominal axial load $P_n = P_u/\phi$, the following nondimensional equation shall be satisfied:

$$(M_{nx}/M_{ox})^K + (M_{ny}/M_{oy})^K \leq 1.0 \tag{4-29}$$

where

M_{nx}, M_{ny} = nominal biaxial bending moments with respect to the x and y axes, respectively

M_{ox}, M_{oy} = uniaxial nominal bending strength at P_n about the x and y axes, respectively

K = 1.5 for rectangular members
= 1.75 for square or circular members
= 1.0 for any member subjected to axial tension

c. M_{ox} and M_{oy} shall be determined in accordance with paragraphs 4-1 through 4-3.

CHAPTER 5

SHEAR

5-1. Shear Strength

The shear strength V_c provided by concrete shall be computed in accordance with ACI 318 except in the cases described in paragraphs 5-2 and 5-3.

5-2. Shear Strength for Special Straight Members

The provisions of this paragraph shall apply only to straight members of box culvert sections or similar structures that satisfy the requirements of 5-2.a and 5-2.b. The stiffening effects of wide supports and haunches shall be included in determining moments, shears, and member properties. The ultimate shear strength of the member is considered to be the load capacity that causes formation of the first inclined crack.

a. Members that are subjected to uniformly (or approximately uniformly) distributed loads that result in internal shear, flexure, and axial compression (but not axial tension).

b. Members having all of the following properties and construction details.

(1) Rectangular cross-sectional shapes.

(2) ℓ_n/d between 1.25 and 9, where ℓ_n is the clear span.

(3) f_c' not more than 6,000 psi.

(4) Rigid, continuous joints or corner connections.

(5) Straight, full-length reinforcement. Flexural reinforcement shall not be terminated even though it is no longer a theoretical requirement.

(6) Extension of the exterior face reinforcement around corners such that a vertical lap splice occurs in a region of compression stress.

(7) Extension of the interior face reinforcement into and through the supports.

c. The shear strength provided the concrete shall be computed as

$$V_c = \left[\left(11.5 - \frac{\ell_n}{d}\right)\sqrt{f_c'}\,\sqrt{1 + \frac{N_u/A_g}{5\sqrt{f_c'}}}\,\right]bd \qquad (5\text{-}1)$$

at a distance of $0.15\ell_n$ from the face of the support.

d. The shear strength provided by the concrete shall not be taken greater than

$$V_c = 2\left[12 - \left(\frac{\ell_n}{d}\right)\right]\sqrt{f_c'}\ bd \tag{5-2}$$

and shall not exceed $10\sqrt{f_c'}$ bd.

5-3. Shear Strength for Curved Members

At points of maximum shear, for uniformly loaded curved cast-in-place members with R/d > 2.25 where R is the radius curvature to the centerline of the member:

$$V_c = \left[4\sqrt{f_c'}\ \sqrt{1 + \frac{N_u/A_g}{4\sqrt{f_c'}}}\ \right]bd \tag{5-3}$$

The shear strength shall not exceed $10\sqrt{f_c'}$ bd.

5-4. Empirical Approach

Shear strength based on the results of detailed laboratory or field tests conducted in consultation with and approved by CECW-ED shall be considered a valid extension of the provisions in paragraphs 5-2 and 5-3.

APPENDIX A

NOTATION

a_d	Depth of stress block at limiting value of balanced condition (Appendix D)
d_d	Minimum effective depth that a singly reinforced member may have and maintain steel ratio requirements (Appendix D)
e'	Eccentricity of axial load measured from the centroid of the tension reinforcement
e'_b	Eccentricity of nominal axial load strength, at balanced strain conditions, measured from the centroid of the tension reinforcement
H_f	Hydraulic structural factor equal to 1.3
k_b	Ratio of stress block depth (a) to the effective depth (d) at balanced strain conditions
k_u	Ratio of stress block depth (a) to the effective depth (d)
K	Exponent, equal to 1.0 for any member subject to axial tension, 1.5 for rectangular members and 1.75 for square or circular members, used in nondimensional biaxial bending expression
ℓ_n	Clear span between supports
M_{DS}	Bending moment capacity at limiting value of balanced condition (Appendix D)
M_{nx}, M_{ny}	Nominal biaxial bending moments with respect to the x and y axes, respectively
M_{ox}, M_{oy}	Uniaxial nominal bending strength at P_n about the x and y axes, respectively
R	Radius of curvature to centerline of curved member

APPENDIX B

DERIVATION OF EQUATIONS FOR FLEXURAL AND AXIAL LOADS

B-1. General

Derivations of the design equations given in paragraphs 4-2 through 4-4 are presented below. The design equations provide a general procedure that may be used to design members for combined flexural and axial load.

B-2. Axial Compression and Flexure

a. Balanced Condition

From Figure B-1, the balanced condition, Equations 4-3 and 4-4 can be derived as follows:

From equilibrium,

$$\frac{P_u}{\phi} = 0.85\ f_c'\ b\ k_u\ d - A_s f_s \tag{B-1}$$

let

$$j_u = d - \frac{a}{2} = d - \frac{k_u d}{2} \tag{B-2}$$

from moment equilibrium,

$$\frac{P_u e'}{\phi} = \left(0.85\ f_c'\ b\ k_u\ d\right)\left(j_u d\right) \tag{B-3}$$

Rewrite Equation B-3 as:

$$\begin{aligned}\frac{P_u e'}{\phi} &= \left(0.85\ f_c'\ b\ k_u\ d\right)\left(d - \frac{k_u d}{2}\right)\\ &= \left(0.85\ f_c'\ bd^2\right)\left(k_u - \frac{k_u^2}{2}\right)\\ &= 0.425\ f_c'\left(2k_u - k_u^2\right) bd^2\end{aligned} \tag{B-4}$$

From the strain diagram at balanced condition (Figure B-1):

$$\frac{c_b}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y}$$

$$\frac{\left(\dfrac{k_b d}{\beta_1}\right)}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_y} \tag{B-5}$$

since $\varepsilon_y = \dfrac{f_y}{E_s}$

$$k_b = \frac{\beta_1 E_s \varepsilon_c}{E_s \varepsilon_c + f_y} \qquad \text{(B-6, Eq. 4-4)}$$

since $e_b' = \dfrac{P_b e'}{P_b}$ (B-7)

e_b' is obtained by substituting Equations B-4 and B-1 into Equation B-7 with $k_u = k_b$, $f_s = f_y$ and $P_u = P_b$.

$$e_b' = \frac{0.425 f_c' (2k_b - k_b^2) bd^2}{0.85 f_c' k_b bd - f_y \rho bd} \tag{B-8}$$

Therefore

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2}{2k_b - \dfrac{f_y \rho}{0.425 f_c'}} \qquad \text{(B-9, Eq. 4-3)}$$

b. Sections Controlled by Tension (Figure B-1).

ϕP_n is obtained from Equation B-1 with $f_s = f_y$ as:

$$\phi P_n = \phi\left(0.85\ f_c'\ b\ k_u d - A_s f_y\right) \quad \text{(B-10, Eq. 4-5)}$$

$$\phi P_n = \phi\left(0.85\ f_c'\ k_u - \rho f_y\right) bd$$

The design moment ϕM_n is expressed as:

$$\phi M_n = \phi P_n e$$

$$\phi M_n = \phi P_n \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] d \quad \text{(B-11)}$$

Therefore,

$$\phi M_n = \phi\left(0.85\ f_c'\ k_u - f_y \rho\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \quad \text{(B-12, Eq. 4-6)}$$

Substituting Equation B-1 with $f_s = f_y$ into Equation B-4 gives

$$\left(0.85\ f_c'\ k_u\ bd - f_y \rho bd\right) e' = 0.425 f_c'\left(2k_u - k_u^2\right) bd^2 \quad \text{(B-13)}$$

which reduces to

$$k_u^2 + 2\left(\frac{e'}{d} - 1\right) k_u - \frac{f_y \rho e'}{0.425 f_c d} = 0 \quad \text{(B-14)}$$

Solving by the quadratic equation:

$$k_u = \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c}\right)\frac{e'}{d}} - \left(\frac{e'}{d} - 1\right) \quad \text{(B-15, Eq. 4-7)}$$

c. Sections Controlled by Compression (Figure B-1)

ϕP_n is obtained from Equation B-1

$$\phi P_n = \phi \left(0.85\ f_c'\ k_u - \rho f_s\right) bd \qquad \text{(B-16, Eq. 4-8)}$$

and ϕM_n is obtained by multiplying Equation B-16 by e.

$$\phi M_n = \phi \left(0.85\ f_c'\ k_u - \rho f_s\right) \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \qquad \text{(B-17)}$$

The steel stress, f_s, is expressed as $f_s = E_s \varepsilon_s$.

From Figure B-1.

$$\frac{c}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s}$$

or

$$\frac{\left(\dfrac{k_u d}{\beta_1}\right)}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s}$$

Therefore,

$$f_s = \frac{E_s \varepsilon_c \left(\beta_1 - k_u\right)}{k_u} \qquad \text{(B-18, Eq. 4-10)}$$

Substituting Equations B-1 and B-18 into B-4 gives

$$0.85\ f_c'\ k_u\ bde' - \left[\frac{E_s\ \varepsilon_c \left(\beta_1 - k_u\right)}{k_u}\right] \rho bde' \qquad \text{(B-19)}$$
$$= 0.425\ f_c' \left(2k_u - k_u^2\right) bd^2$$

which can be arranged as

$$k_u^3 + 2\left(\frac{e'}{d} - 1\right) k_u^2 + \left(\frac{E_s \varepsilon_c \rho e'}{0.425 f_c' d}\right) k_u - \frac{\beta_1 E_s \varepsilon_c \rho e'}{0.425 f_c' d} = 0 \qquad \text{(B-20, Eq. 4-11)}$$

B-3. Flexural and Compressive Capacity-Tension and Compression Reinforcement (Figure B-2)

a. Balanced Condition

Using Figure B-2, the balanced condition, Equation 4-13 can be derived as follows:

From equilibrium,

$$\frac{P_u}{\phi} = 0.85\ f_c'\ k_u bd + f_s'\ \rho' bd - f_s \rho bd \qquad \text{(B-21)}$$

In a manner similar to the derivation of Equation B-4, moment equilibrium results in

$$\frac{P_u e'}{\phi} = 0.425\ f_c'\ (2k_u - k_u^2)\ bd^2 + f_s'\ \rho' bd(d - d') \qquad \text{(B-22)}$$

As in Equation B-6,

$$k_b = \frac{\beta_1 E_s \varepsilon_c}{E_s \varepsilon_c + f_y} \qquad \text{(B-23)}$$

$$\text{since } e_b' = \frac{P_b e'}{P_s} \qquad \text{(B-24)}$$

and using Equations B-21 and B-22:

$$e_b' = \frac{0.425\ f_c'\ (2k_b - k_b^2) bd^2 + f_s'\ \rho' bd(d - d')}{0.85\ f_c\ k_b bd + f_s\ \rho' bd - f_s\ \rho bd} \qquad \text{(B-25)}$$

which can be rewritten as

$$e_b' = \frac{(2k_b - k_b^2)d + \dfrac{f_s'\rho'}{0.425 f_c'}(d - d')}{2k_b + \dfrac{f_s\rho'}{0.425 f_c'} - \dfrac{f_y\rho}{0.425 f_c'}}$$

or

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2 + \dfrac{f_s'\rho'\left(1 - \dfrac{d'}{d}\right)}{0.425 f_c'}}{2k_b - \dfrac{f_y\rho}{0.425 f_c'} + \dfrac{f_s\rho'}{0.425 f_c'}} \qquad \text{(B-26, Eq. 4-13)}$$

b. Sections Controlled by Tension (Figure B-2)

ϕP_n is obtained as Equation B-21 with $f_s = f_y$.

$$\phi P_n = \phi\left(0.85\, f_c'\, k_u + \rho' f_s' - \rho f_y\right) bd \qquad \text{(B-27, Eq. 4-14)}$$

Using Equations B-11 and B-27,

$$\phi M_n = \phi\left(0.85\, f_c'\, k_u + \rho' f_s' - \rho f_y\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \qquad \text{(B-28, Eq. 4-15)}$$

From Figure B-2

$$\frac{\varepsilon_s'}{c - d'} = \frac{\varepsilon_y}{d - c}\ ;\ f_s' = E_s\varepsilon_s'\ ;\ c = \frac{k_u d}{\beta_1}$$

Therefore,

$$\frac{f_s'}{E_s} = \left(\frac{k_u d}{\beta_1} - d'\right)\left(\frac{\varepsilon_y}{d - \dfrac{k_u d}{\beta_1}}\right)$$

or

$$f_s' = \frac{\left(k_u - \beta_1 \dfrac{d'}{d}\right)}{\left(\beta_1 - k_u\right)} E_s \varepsilon_y \qquad \text{(B-29, Eq. 4-16)}$$

Substituting Equation B-21 with $f_s = f_y$ into Equation B-22 gives,

$$\begin{aligned}&\left(0.85\ f_c'\ k_u bd + f_s'\rho' bd - f_y \rho bd\right)e' \\ &\quad = 0.425\ f_c'\ \left(2k_u - k_u^2\right)bd^2 + f_s'\rho' bd(d - d')\end{aligned} \qquad \text{(B-30)}$$

Using Equation B-29, Equation B-30 can be written as:

$$\begin{aligned}&k_u^3 + \left[2\left(\frac{e'}{d} - 1\right) - \beta_1\right]k_u^2 - \left\{\frac{f_y}{0.425 f_c'}\left[\rho'\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right]\right. \\ &\left. + 2\ \beta_1 \left(\frac{e'}{d} - 1\right)\right\}k_u + \frac{f_y \beta_1}{0.425 f_c'}\left[\rho'\frac{d'}{d}\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right] \\ &= 0\end{aligned} \qquad \text{(B-31, Eq. 4-17)}$$

c. Sections Controlled by Compression (Figures B-2)

ϕP_n is obtained from equilibrium

$$\phi P_n = \phi\ \left(0.85\ f_c'\ k_u + \rho' f_s' - \rho f_s\right)\ bd \qquad \text{(B-32, Eq. 4-18)}$$

Using Equations B-11 and B-32,

$$\phi M_n = \phi \left(0.85\ f_c'\ k_u + \rho' f_s' - \rho f_s\right) \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \qquad \text{(B-33, Eq. 4-19)}$$

From Figure B-2

$$\frac{\varepsilon_s}{d - c} = \frac{\varepsilon_c}{c}\ ;\quad f_s = E_s \varepsilon_s\ ;\quad c = \frac{k_u d}{\beta_1}$$

which can be written as

$$f_s = \frac{E_s \varepsilon_c (\beta_1 - k_u)}{k_u} \qquad \text{(B-34, Eq. 4-20)}$$

Also,

$$\frac{\varepsilon_s'}{c - d'} = \frac{\varepsilon_c}{c}$$

which can be rewritten as

$$f_s' = \frac{E_s \varepsilon_c \left[k_u - \beta_1 \left(\frac{d'}{d}\right)\right]}{k_u} \qquad \text{(B-35, Eq. 4-21)}$$

From Equations B-21 and B-22

$$\begin{aligned} &\left(0.85 f_c'\ k_u bd = f_s' \rho' bd - f_s \rho bd\right) e' \\ &\quad = 0.425\ f_c' \left(2k_u - k_u^2\right) bd^2 + f_s'\ \rho' bd(d - d') \end{aligned} \qquad \text{(B-36)}$$

Substituting Equations B-34 and B-35 with $k_b = k_u$ into Equation B-36 gives

$$k_u^3 + 2\left(\frac{e'}{d} - 1\right)k_u^2 + \frac{E_s\varepsilon_c}{0.425f_c}\left[(\rho + \rho')\left(\frac{e'}{d}\right) - \rho'\left(1 - \frac{d'}{d}\right)\right]k_u$$

$$- \frac{\beta_1 E_s\varepsilon_c}{0.425f_c}\left[\rho'\left(\frac{d'}{d}\right)\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \rho\left(\frac{e'}{d}\right)\right] = 0 \qquad \text{(B-37, Eq. 4-22)}$$

B-4. Flexural and Tensile Capacity

a. Pure Tension (Figure B-3)

From equilibrium (double reinforcement)

$$\phi P_n = \phi(A_s + A_s')f_y \qquad \text{(B-38)}$$

For design, the axial load strength of tension members is limited to 80 percent of the design axial load strength at zero eccentricity.

Therefore,

$$\phi P_{n(\max)} = 0.8\phi\,(\rho + \rho')\,f_y\,bd \qquad \text{(B-39, Eq. 4-23)}$$

b. For the case where $1 - \frac{h}{2d} \geq \frac{e'}{d} \geq 0$, the applied tensile resultant $\frac{P_u}{\phi}$ lies between the two layers of steel.

From equilibrium

$$\phi P_n = \phi\,(A_s f_y + A_s' f_s')$$

or

$$\phi P_n = \phi\,(\rho f_y + \rho' f_s')bd \qquad \text{(B-40, Eq. 4-24)}$$

and

$$\phi M_n = \phi\,P_n\left[\left(1 - \frac{h}{2d}\right) - \frac{e'}{d}\right]d$$

or

$$\phi M_n = \phi\left(\rho f_y + \rho' f_s'\right)\left[\left(1 - \frac{h}{2d}\right) - \frac{e'}{d}\right] bd^2 \qquad \text{(B-41, Eq. 4-25)}$$

From Figure B-3,

$$\frac{\varepsilon_s'}{a + d'} = \frac{\varepsilon_y}{a + d}$$

which can be rewritten as

$$f_s' = f_y \frac{\left(k_u + \frac{d'}{d}\right)}{\left(k_u + 1\right)} \qquad \text{(B-42, Eq. 4-26)}$$

From Figure B-3 equilibrium requires:

$$A_s f_s e' = A_s' f_s' \left(d - d' - e'\right) \qquad \text{(B-43)}$$

Substituting Equation B-42 and $f_s = f_y$ into Equation B-43 results in

$$k_u = \frac{\rho'\left(\frac{d'}{d}\right)\left(1 - \frac{d'}{d} - \frac{e'}{d}\right) - \rho\left(\frac{e'}{d}\right)}{\rho\left(\frac{e'}{d}\right) - \rho'\left(1 - \frac{d'}{d} - \frac{e'}{d}\right)} \qquad \text{(B-44, Eq. 4-27)}$$

c. The case where $(e'/d) < 0$ is similar to the combined flexural and compression case. Therefore, k_u is derived in a manner similar to the derivation of Equation B-15 and is given as

$$k_u = -\left(\frac{e'}{d} - 1\right) - \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c}\right)\frac{e'}{d}} \qquad \text{(B-45, Eq. 4-28)}$$

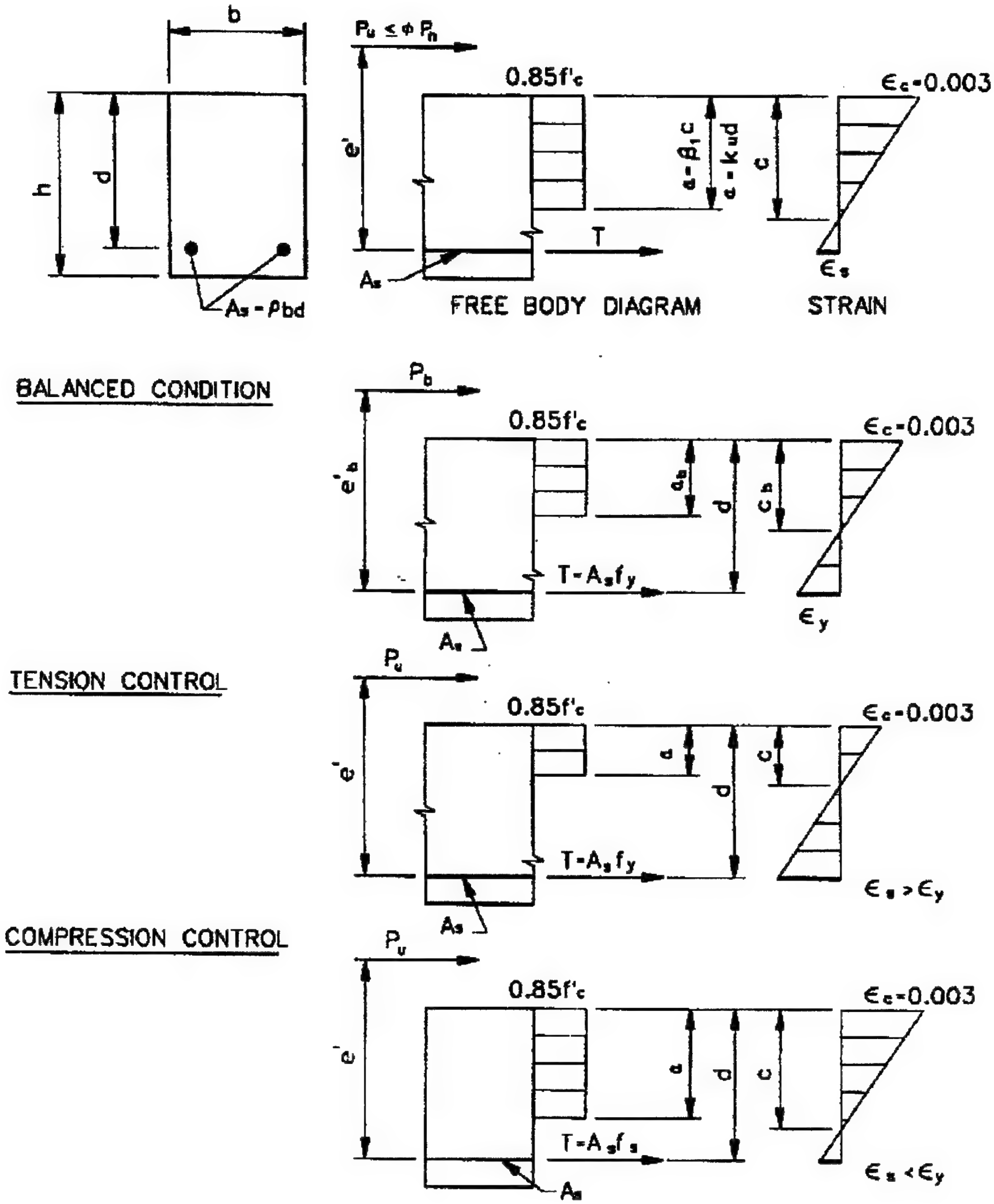


Figure B-1. Axial compression and flexure, single reinforcement

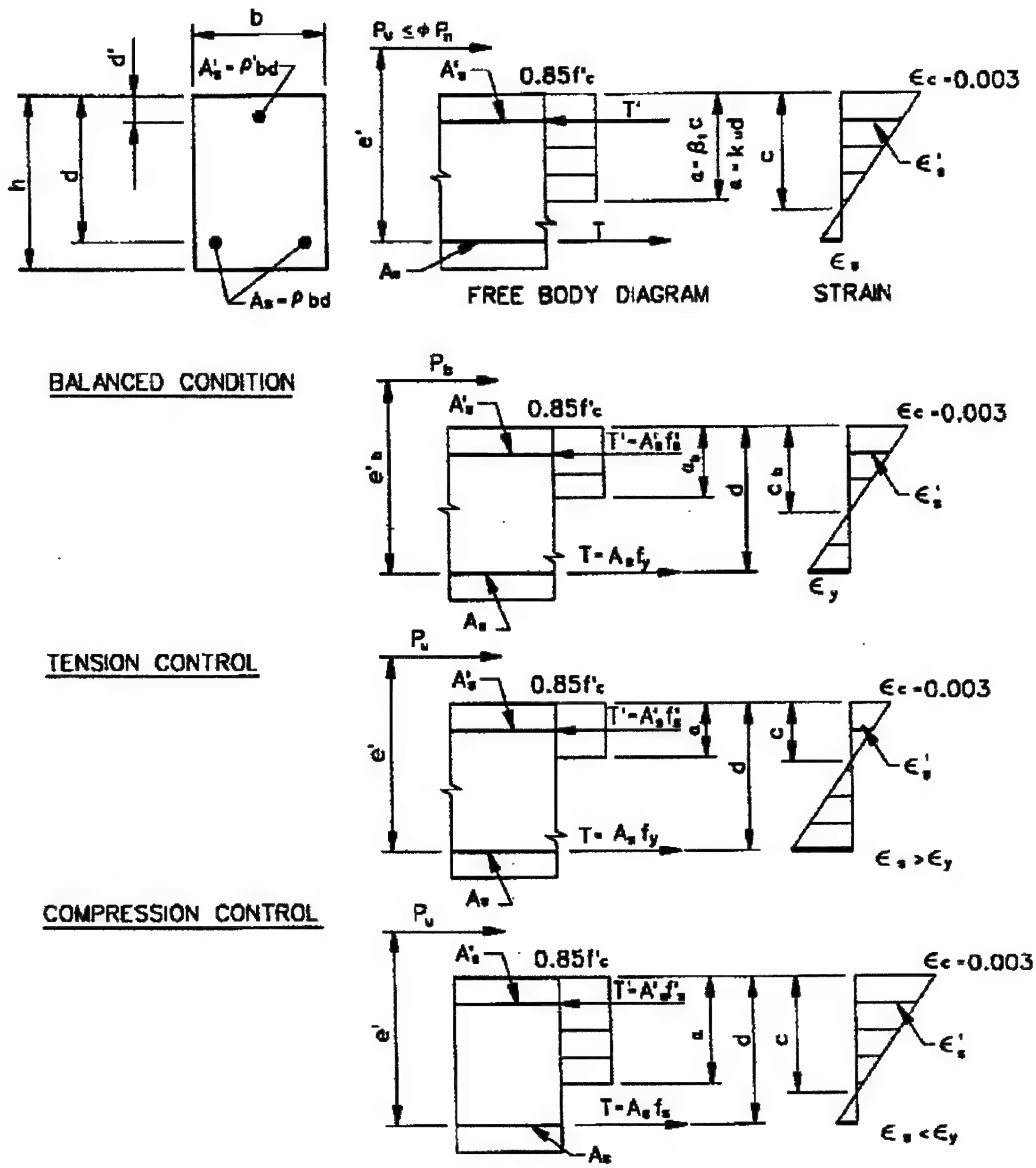


Figure B-2. Axial compression and flexure, double reinforcement

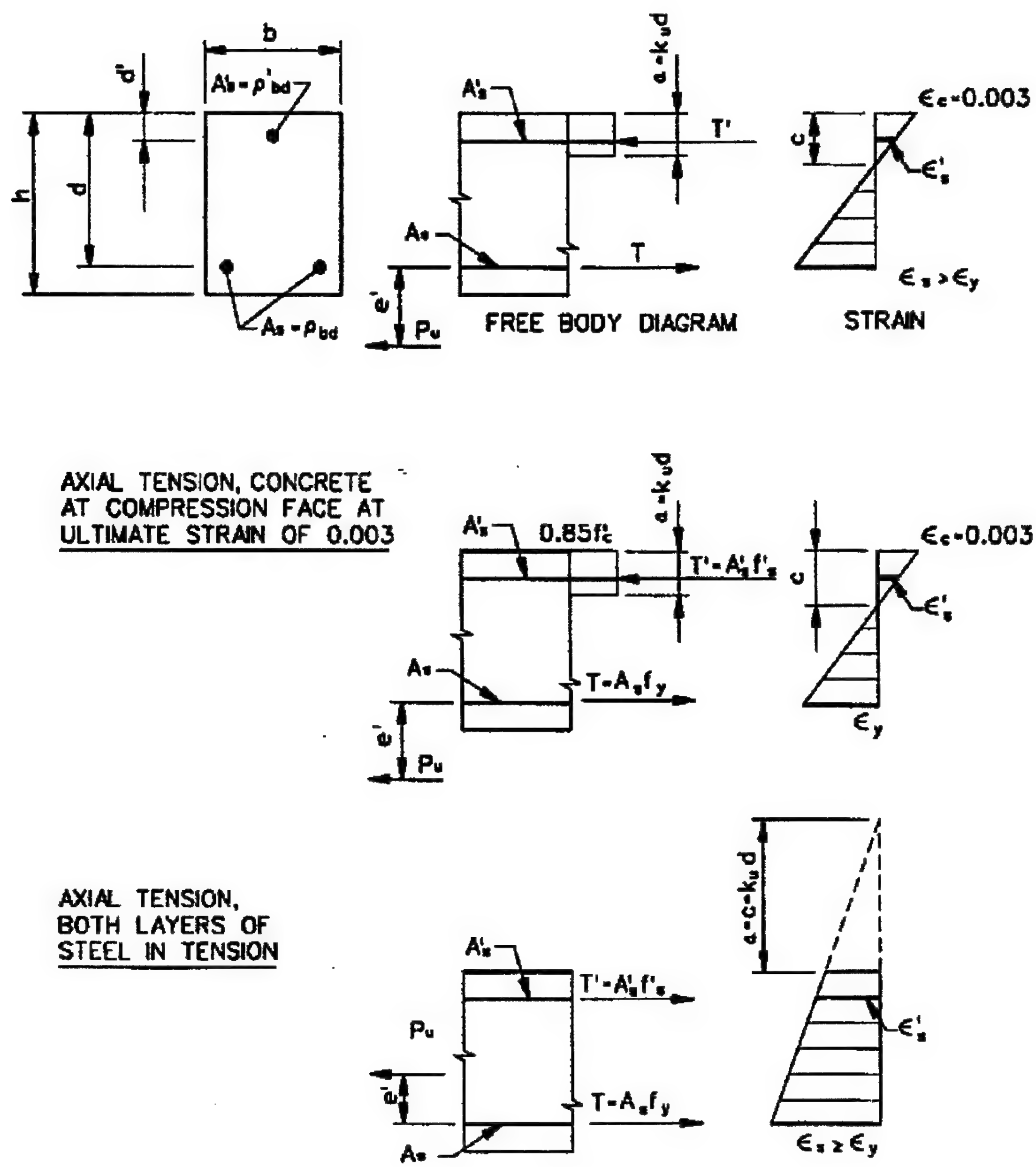


Figure B-3. Axial tension and flexure, double reinforcement

APPENDIX C

INVESTIGATION EXAMPLES

C-1. General

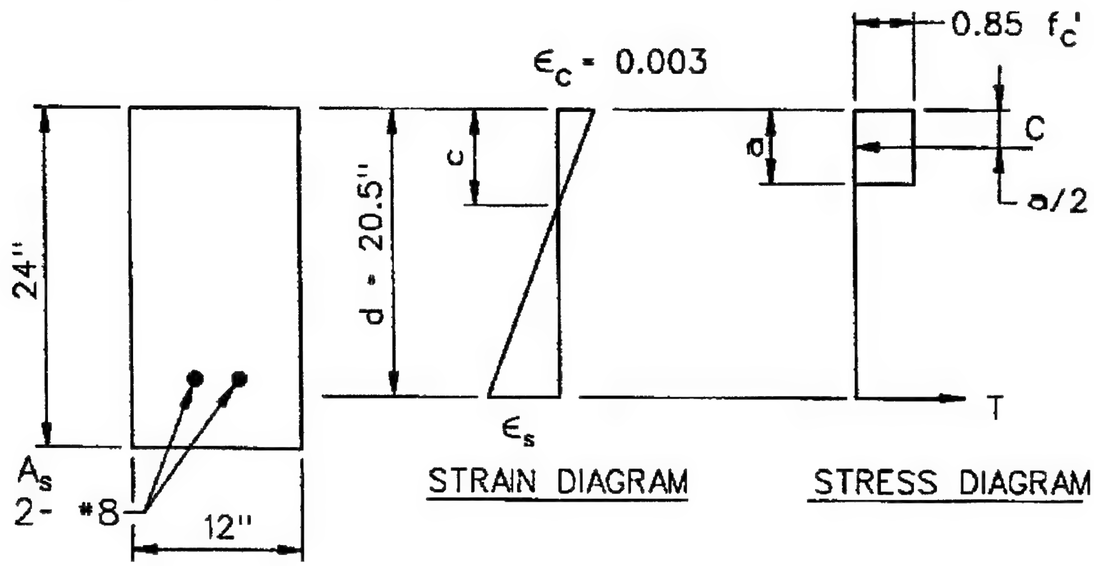
For the designer's convenience and reference, the following examples are provided to illustrate how to determine the flexural capacity of existing concrete sections in accordance with this Engineer Manual and ACI 318.

C-2. Analysis of a Singly Reinforced Beam

Given: $f_c' = 3$ ksi $\qquad \beta_1 = 0.85$

$f_y = 60$ ksi $\qquad E_s = 29{,}000$ ksi

$A_s = 1.58$ in.^{2}



Solution:

1. Check steel ratio

$$\rho_{act} = \frac{A_s}{bd}$$

$$= \frac{1.58}{12(20.5)}$$

$$= 0.006423$$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \left(\frac{87,000}{87,000 + f_y}\right)$$

$$= 0.85\ (0.85) \left(\frac{3}{60}\right) \left(\frac{87,000}{87,000 + 60,000}\right)$$

$$= 0.02138$$

in accordance with Paragraph 3-5 check:

$$0.25\rho_b = 0.00534$$

$$0.375\rho_b = 0.00802$$

$$\rho_{act} = 0.00642$$

$$0.25\rho_b < \rho_{act} < 0.375\rho_b$$

ρ_{act} is greater than the recommended limit, but less than the maximum permitted upper limit not requiring special study or investigation. Therefore, no special consideration for serviceability, constructibility, and economy is required. This reinforced section is satisfactory.

2. Assume the steel yields and compute the internal forces:

$$T = A_s f_y = 1.58\ (60) = 94.8 \text{ kips}$$

$$C = 0.85\ f_c' ba$$

$$C = 0.85\ (3)\ (12) a = 30.6a$$

3. From equilibrium set $T = C$ and solve for a:

$$94.8 = 30.6a \longrightarrow a = 3.10 \text{ in.}$$

$$\text{Then, } a = \beta_1 c \longrightarrow c = \frac{3.10}{0.85} = 3.65 \text{ in.}$$

4. Check ε_s to demonstrate steel yields prior to crushing of the concrete:

$$\frac{\varepsilon_s}{20.5 - c} = \frac{0.003}{c}$$

$$\varepsilon_s = 16.85\left(\frac{0.003}{3.65}\right) = 0.0138$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$\varepsilon_s > \varepsilon_y$ Ok, steel yields

5. Compute the flexural capacity:

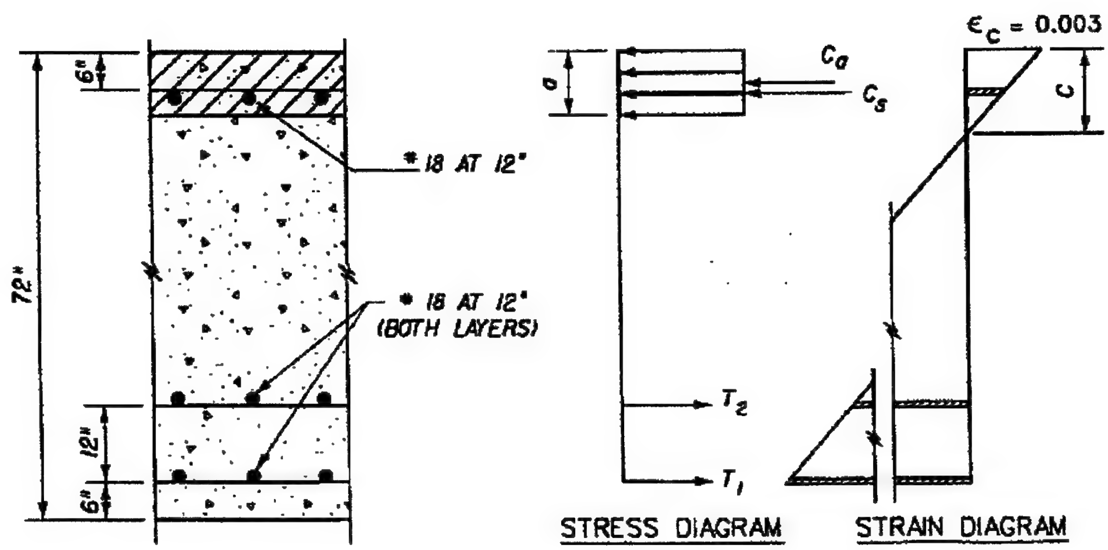
$$\phi M_n = \phi\,(A_s f_y)\;(d - a/2)$$

$$= 0.90\;(94.8)\left(20.5 - \frac{3.10}{2}\right)$$

$$= 1616.8 \text{ in.-k}$$

$$= 134.7 \text{ ft-k}$$

C-3. Analysis of an Existing Beam - Reinforcement in Both Faces



Given: $f_c' = 3{,}000$ psi $\quad \varepsilon_c = 0.003$

$f_y = 60{,}000$ psi $\quad \beta_1 = 0.85$

$A_s = 8.00$ in.^{2} $\quad E_s = 29{,}000{,}000$ psi

$A_s' = 4.00$ in.^{2}

Solution:

1. First analyze considering steel in tension face only

$$\rho = \frac{A_s}{bd} = \frac{8}{(60)(12)} = 0.011$$

$$\rho_{bal} = 0.85\frac{\beta_1 f_c'}{f_y} = \frac{87{,}000}{87{,}000 + f_y} = 0.0214$$

$$\rho = \frac{0.011}{0.0214}\rho_{bal} = 0.51\rho_b$$

Note: ρ exceeds maximum permitted upper limit not requiring special study or investigation = $0.375\ \rho_b$. See Chapter 3.

$$T = A_s f_y$$

$$T = 8(60) = 480 \text{ kips}$$

then $C_c = 0.85 f_c' ba = 30.6a$

$$T = C_c$$

$\therefore\ a = 15.7$ in. and $c = 18.45$ in.

By similar triangles, demonstrate that steel yields

$$\frac{\varepsilon_c}{18.45} = \frac{\varepsilon_{s(2)}}{54 - c} \Rightarrow \varepsilon_{s(2)} = 0.0057 > \varepsilon_y = 0.0021$$

ok; both layers of steel yield.

Moment capacity = 480 kips (d - a/2)

= 480 kips (52.15 in.)

M = 25,032 in.-k

2. Next analyze considering steel in compression face

$$\rho' = \frac{4}{12(60)} = 0.0056$$

$$\rho - \rho' = 0.0054$$

$$= 0.85\frac{\beta_1 f_c'}{f_y} \cdot \frac{d'}{d}\left(\frac{87{,}000}{87{,}000 - f_y}\right) = 0.016$$

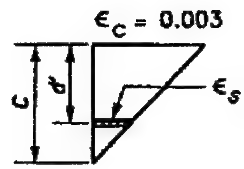
$\rho - \rho' \leq 0.0116$ $\therefore$ compression steel does not yield, must do general analysis using $\sigma : \varepsilon$ compatability

Locate neutral axis

$T = 480$ kips

$C_c = 0.85f_c'ba = 30.6a$

$C_s = A_s' \, (f_s' - 0.85f_c') = 4(f_s' - 2.55)$



By similar triangles

$$\frac{\varepsilon_s'}{c - 6} = \frac{0.003}{c}$$

Substitute $c = \dfrac{a}{0.85} = 1.176a$

Then $\varepsilon_s' = 0.003 - \dfrac{0.0153}{a}$

Since $f_s' = E\varepsilon_s' \Rightarrow f_s' = \left(87 - \dfrac{443.7}{a}\right)$ksi

Then

$$C_s = 4\left(87 - 2.55 - \frac{443.7}{a}\right)\text{kips}$$

$T = C_c + C_s = 480$ kips

Substitute for C_c and C_s and solve for a

$$30.6a + 337.8 - \frac{1774.8}{a} = 480$$

$$a^2 - 4.65a - 58 = 0$$

Then $a = 10.3$ in.

and $c = 12.1$ in.

Check $\varepsilon_s' > \varepsilon_y$

By similar triangles $\frac{0.003}{12.1} = \frac{\varepsilon_s'}{d - 12.1}$

$$\varepsilon_s' = 0.0119 > 0.0021$$

$C_c = 30.6a \approx 315$ kips

$C_s = 4(41.37) \approx 165$ kips

$C_c + C_s = 480 \text{ kips} = T$

$$\text{Resultant of } C_c \text{ and } C_s = \frac{315\left(\frac{10.3}{2}\right) + (165)(6)}{480} = 5.4 \text{ in.}$$

Internal Moment Arm = 60 − 5.4 = 54.6 in.

$M = 480(54.6) = 26{,}208$ in.-k

	Comparison	
	Tension Steel Only	Compression Steel
a	15.7 in.	10.3 in.
c	18.45 in.	12.1 in.
Arm	52.15 in.	54.6 in.
M	25,032 in.-k	26,208 in.-k ⇒ 4.7 percent increase

APPENDIX D

DESIGN EXAMPLES

D-1. Design Procedure

For convenience, a summary of the steps used in the design of the examples in this appendix is provided below. This procedure may be used to design flexural members subjected to pure flexure or flexure combined with axial load. The axial load may be tension or compression.

Step 1 - Compute the required nominal strength M_n, P_n where M_u and P_u are determined in accordance with paragraph 4-1.

$$M_n = \frac{M_u}{\phi} \qquad P_n = \frac{P_u}{\phi}$$

Note: Step 2 below provides a convenient and quick check to ensure that members are sized properly to meet steel ratio limits. The expressions in Step 2a are adequate for flexure and small axial load. For members with significant axial loads the somewhat more lengthy procedures of Step 2b should be used.

Step 2a - Compute d_d from Table D-1. The term d_d is the minimum effective depth a member may have and meet the limiting requirements on steel ratio. If $d \geq d_d$ the member is of adequate depth to meet steel ratio requirements and A_s is determined using Step 3.

Step 2b - When significant axial load is present, the expressions for d_d become cumbersome and it becomes easier to check the member size by determining M_{DS}. M_{DS} is the maximum bending moment a member may carry and remain within the specified steel ratio limits.

$$M_{DS} = 0.85f_c' \, a_d b(d - a_d/2) - (d - h/2)P_n \qquad \text{(D-1)}$$

where

$$a_d = K_d d \qquad \text{(D-2)}$$

and K_d is found from Table D-1.

Step 3 - Singly Reinforced - When $d \geq d_d$ (or $M_n \leq M_{DS}$) the following equations are used to compute A_s.

$$K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f_c' b d^2}} \tag{D-3}$$

$$A_s = \frac{0.85 f_c' K_u b d - P_n}{f_y} \tag{D-4}$$

Table D-1

Minimum Effective Depth

f_c' (psi)	f_y (psi)	$\frac{\rho^*}{\rho_b}$	K_d	d_d (in.)
3000	60	0.25	0.125765	$\sqrt{\frac{3.3274M_n**}{b}}$
4000	60	0.25	0.125765	$\sqrt{\frac{2.4956M_n*}{b}}$
5000	60	0.25	0.118367	$\sqrt{\frac{2.1129M_n*}{b}}$

* See Section 3-5. Maximum Tension Reinforcement

** M_n units are inch-kips.

where

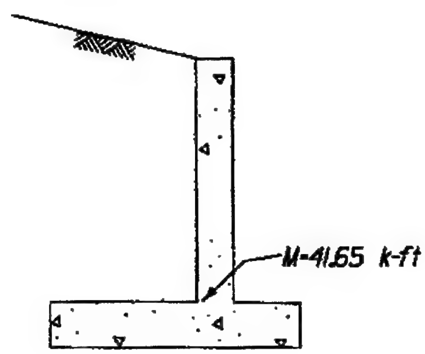
$$K_d = \frac{\left(\frac{\rho}{\rho_b}\right)\beta_1 \varepsilon_c}{\varepsilon_c + \frac{f_y}{E_s}}$$

$$d_d = \sqrt{\frac{M_n}{0.85 f_c' k_d b \left(1 - \frac{k_d}{2}\right)}}$$

D-2. Singly Reinforced Example

The following example demonstrates the use of the design procedure outlined in paragraph D-1 for a Singly Reinforced Beam with the recommended steel ratio of 0.25 ρ_b. The required area of steel is computed to carry the moment at the base of a retaining wall stem.

Given: M = 41.65 k-ft
(where M = moment from <u>unfactored</u> dead and live loads)
f_c' = 3.0 ksi
f_y = 60 ksi
d = 20 in.



First compute the required strength, M_u.

$$M_u = 1.7\ H_f(D + L)$$

$$M_u = (1.7)\ (1.3)\ (41.65) = 92.047 \text{ k-ft}$$

<u>Step 1</u>. $M_n = M_u/\phi = 92.047/0.90 = 102.274$ k-ft

Step 2. $d_d = \sqrt{\dfrac{3.3274M_n}{b}} = 18.45$ in. (Table D-1)

$d > d_d$ therefore member size is adequate

Step 3. $$K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f_c' b d^2}} \quad \text{(D-3)}$$

$$K_u = 1 - \sqrt{1 - \frac{(102.274)(12)}{(0.425)(3.0)(12)(20)^2}} = 0.10587$$

$$A_s = \frac{0.85 f_c' K_u b d}{f_y} = \frac{(0.85)(3.0)(0.10587)(12)(20)}{60} \quad \text{(D-4)}$$

$A_s = 1.08$ sq in.

D-3. Combined Flexure Plus Axial Load Example

The following example demonstrates the use of the design procedure outlined in paragraph D-1 for a beam subjected to flexure plus small axial compressive load. The amount of tensile steel required to carry the moment and axial load at the base of a retaining wall stem is found.

Given: $M = 41.65$ k-ft

$P = 5$ kips (weight of stem)

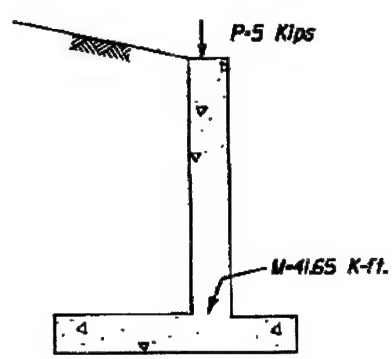
where M and P are the moment and axial load from an unfactored analysis.

$f_c' = 3.0$ ksi

$f_y = 60$ ksi

d = 20 in.

h = 24 in.



First compute the required strength, M_u, P_u

$$M_u = 1.7\ H_f(D + L)$$

$$M_u = (1.7)(1.3)(41.65) = 92.047\ k\text{-}ft$$

$$P_u = 1.7\ H_f(D + L)$$

$$P_u = (1.7)(1.3)(5.0) = 11.05 \text{ kips}$$

Since axial load is present a value must be found for ϕ.

For small axial load $\phi \cong 0.9 - [(0.20\ P_u)/(0.10 f_c^{'} A_g)]$

$$\phi \cong 0.88$$

Step 1. $M_n = M_u/\phi = 92.047/0.88 = 104.60\ k\text{-}ft$

$$P_n = P_u/\phi = 11.05/0.88 = 12.56 \text{ kips}$$

Step 2. $a_d = K_d d$ (D-2)

$$a_d = (0.12577)(20) = 2.515$$

$$M_{DS} = 0.85 f_c^{'}\ a_d\ b(d - a_d/2.0) - (d - h/2.0)P_n \qquad \text{(D-1)}$$

$$M_{DS} = (0.85)(3.0)(2.515)(12)(20 - 1.258) - (20 - 12)(12.56)$$

$$M_{DS} = 1341.9\ k\text{-}in. \text{ or } 111.82\ k\text{-}ft$$

$M_{DS} > M_n$ therefore member size is adequate

Step 3. $K_u = 1 - \sqrt{1 - \dfrac{M_n + P_n(d - h/2)}{0.425 f_c' bd^2}}$

$$K_u = 1 - \sqrt{1 - \frac{(12)104.6 + 12.56(20 - 12)}{(0.425)(3.0)(12)(20)^2}}$$

$$K_u = 0.11768$$

$$A_s = \frac{0.85 f_c' K_u bd - P_n}{f_y}$$

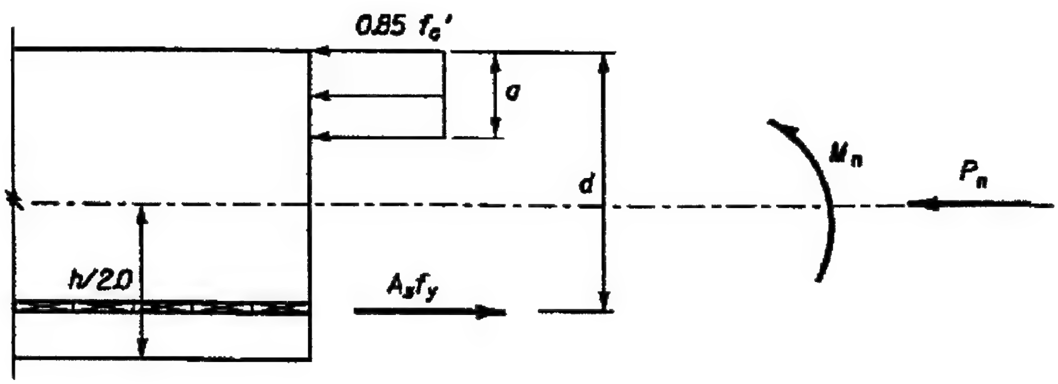
$$A_s = \frac{(0.85)(3.0)(0.11768)(12)(20) - 12.56}{60} \qquad \text{(D-4)}$$

$$A_s = 0.99 \text{ sq in.}$$

D-4. Derivation of Design Equations

The following paragraphs provide derivations of the design equations presented in paragraph D-1.

(1) Derivation of Design Equations for Singly Reinforced Members. The figure below shows the conditions of stress on a singly reinforced member subjected to a moment M_n and load P_n. Equations for design may be developed by satisfying conditions of equilibrium on the section.



By requiring the ΣM about the tensile steel to equal zero

$$M_n = 0.85 f_c' \, ab(d - a/2) - P_n(d - h/2) \qquad \text{(D-5)}$$

By requiring the ΣH to equal zero

$$A_s f_y = 0.85 f_c' ab - P_n \tag{D-6}$$

Expanding Equation D-5 yields

$$M_n = 0.85 f_c' abd - 0.425 f_c' a^2 b - P_n(d - h/2)$$

Let $a = K_u d$ then

$$M_n = 0.85 f_c' K_u bd^2 - 0.425 f_c' K_u^2 d^2 b - P_n(d - h/2)$$

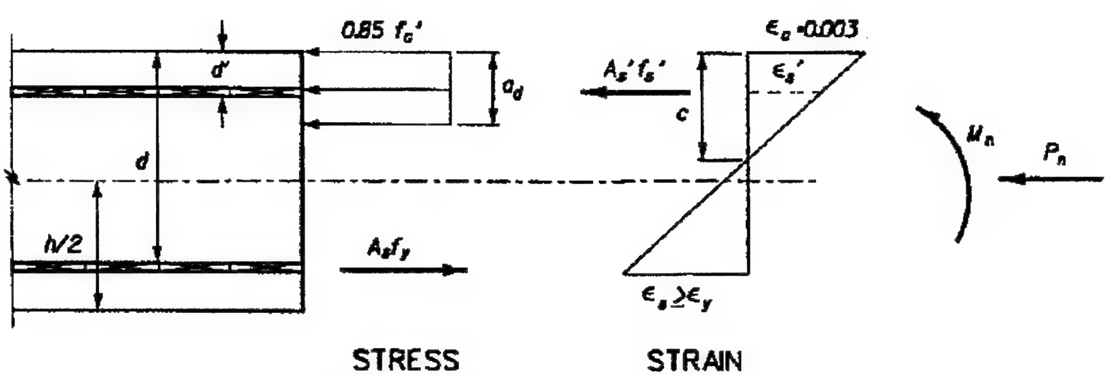
The above equation may be solved for K_u using the solution for a quadratic equation

$$K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f_c bd^2}} \tag{D-3}$$

Substituting $K_u d$ for a in Equation D-6 then yields

$$A_s = \frac{0.85 f_c' K_u bd - P_n}{f_y}$$

(2) Derivation of Design Equations for Doubly Reinforced Members. The figure below shows the conditions of stress and strain on a doubly reinforced member subjected to a moment M_n and load P_n. Equations for design are developed in a manner identical to that shown previously for singly reinforced beams.



Requiring ΣH to equal zero yields

$$A_s = \frac{0.85 f_c' K_d b d - P_n + A_s' f_s'}{f_y} \tag{D-7}$$

By setting $a_d = \beta_1 c$ and using the similar triangles from the strain diagram above, ε_s' and f_s' may be found:

$$f_s' = \frac{(a_d - \beta_1 d')\varepsilon_c E_s}{a_d}$$

An expression for the moment carried by the concrete (M_{DS}) may be found by summing moments about the tensile steel of the concrete contribution.

$$M_{DS} = 0.85 f_c' a_d b (d - a_d/2) - (d - h/2) P_n \tag{D-1}$$

Finally, an expression for A_s' may be found by requiring the compression steel to carry any moment above that which the concrete can carry (M_n - M_{DS}).

$$A_s' = \frac{M_n - M_{DS}}{f_s'(d - d')} \tag{D-8}$$

(3) Derivation of Expression of d_d. The expression for d_d is found by substituting $a_d = k_d d_d$ in the equation shown above for M_{DS} and solving the resulting quadratic expression for d_d.

$$d_d = \sqrt{\frac{M_{DS}}{\left[0.85 f_c' K_d b (1 - K_d/2)\right]}} \tag{D-9}$$

D-5. Shear Strength Example for Special Straight Members

Paragraph 5.2 describes the conditions for which a special shear strength criterion shall apply for straight members. The following example demonstrates the application of Equation 5-1. Figure D-1 shows a rectangular conduit with factored loads, 1.7 H_f (dead load + live Load). The following parameters are given or computed for the roof slab of the conduit.

$f_c' = 4{,}000$ psi

$\ell_n = 10.0$ ft = 120 in.

$d = 2.0$ ft = 24 in.

$b = 1.0$ ft (unit width) = 12 in.

$N_u = 6.33(5) = 31.7$ kips

$A_g = 2.33$ sq ft = 336 sq in.

$$V_c = \left[\left(11.5 - \frac{120 \text{ in.}}{24 \text{ in.}}\right)\sqrt{4{,}000}\sqrt{1 + \left(\frac{\frac{31{,}700 \text{ lb}}{336 \text{ sq in.}}}{5\sqrt{4{,}000}}\right)}\right](12 \text{ in.})(24 \text{ in.}) \qquad \text{(D-10, Eq. 5-1)}$$

$V_c = 134{,}906$ lb = 134.9 kips

Check limit $V_c = 10\sqrt{f_c'}\,bd = 10\sqrt{4{,}000}$ (12 in.)(24 in.) = 182,147 lb

Compare shear strength with applied shear.

$\phi V_c = 0.85(134.9 \text{ kips}) = 114.7$ kips

V_u at $0.15(\ell_n)$ from face of the support is

$$V_u = w\left(\frac{\ell_n}{2} - 0.15\,\ell_n\right)$$

$$= 15.0 \text{ kips/ft}\left[\left(\frac{10 \text{ ft}}{2}\right) - (0.15)(10 \text{ ft})\right]$$

$$= 52.5 \text{ kips} < \phi V_c \text{ ; shear strength adequate}$$

D-6. Shear Strength Example for Curved Members

Paragraph 5-3 describes the conditions for which Equation 5-3 shall apply. The following example applies Equation 5-3 to the circular conduit presented in Figure D-2. Factored loads are shown, and the following values are given or computed:

$f'_c = 4{,}000$ psi

$b = 12$ in.

$d = 43.5$ in.

$A_g = 576$ sq in.

$N_u = 162.5$ kips

$V_u = 81.3$ kips at a section 45 degrees from the crown

$$V_c = 4\sqrt{4{,}000}\left[\sqrt{1 + \left(\frac{\frac{162{,}500 \text{ lb}}{576 \text{ sq in.}}}{4\sqrt{4{,}000}}\right)}\right](12 \text{ in.})(43.5 \text{ in.})$$

$V_c = 192{,}058$ lb $= 192.1$ kips

Check limit $V_c = 10\sqrt{f'_c}\ bd = 10\sqrt{4{,}000}\ (12 \text{ in.})(43.5 \text{ in.}) = 330{,}142$ lb

Compare shear strength with applied shear

$\phi V_c = 0.85(192.1 \text{ kips}) = 163.3$ kips

$V_u < \phi V_c$; shear strength adequate

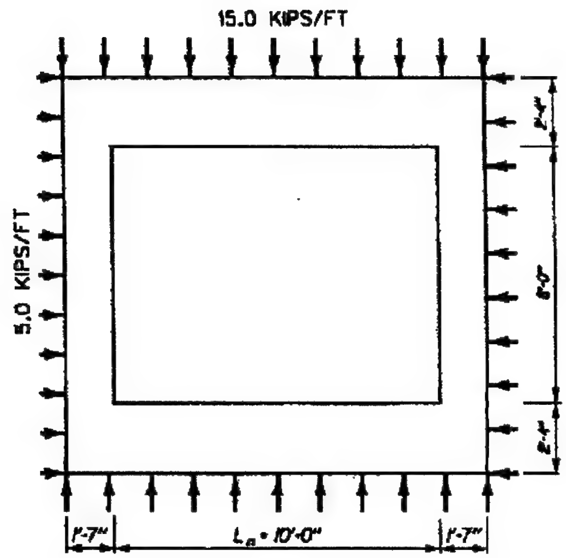


Figure D-1. Rectangular conduit

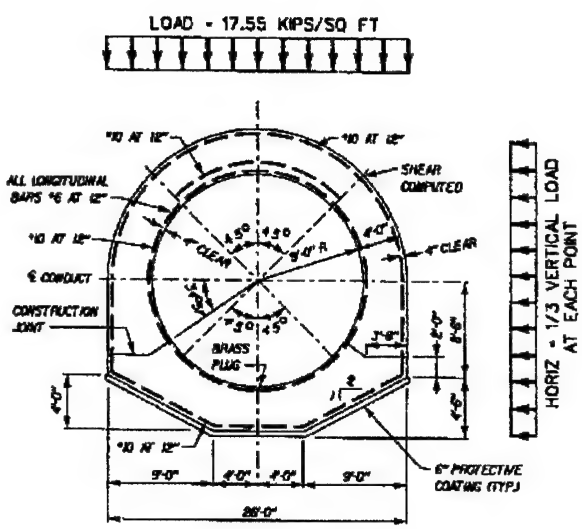


Figure D-2. Circular conduit

APPENDIX E

INTERACTION DIAGRAM

E-1. Introduction

A complete discussion on the construction of interaction diagrams is beyond the scope of this manual; however, in order to demonstrate how the equations presented in Chapter 4 may be used to construct a diagram a few basic points will be computed. Note that the effects of ϕ, the strength reduction factor, have not been considered. Using the example cross section shown below compute the points defined by 1, 2, 3 notations shown in Figure E-1.

Given: $f_c' = 3.0$ ksi

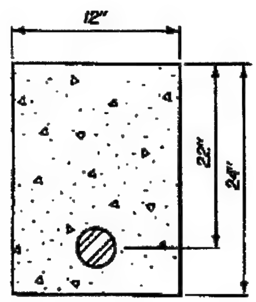
$f_y = 60$ ksi

$A_s = 2.0$ sq in.

$d = 22$ in.

$h = 24$ in.

$b = 12$ in.



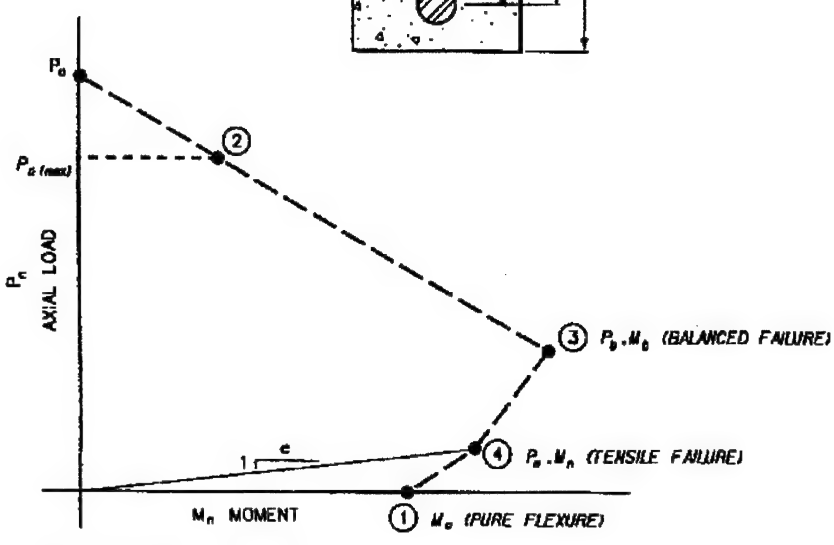
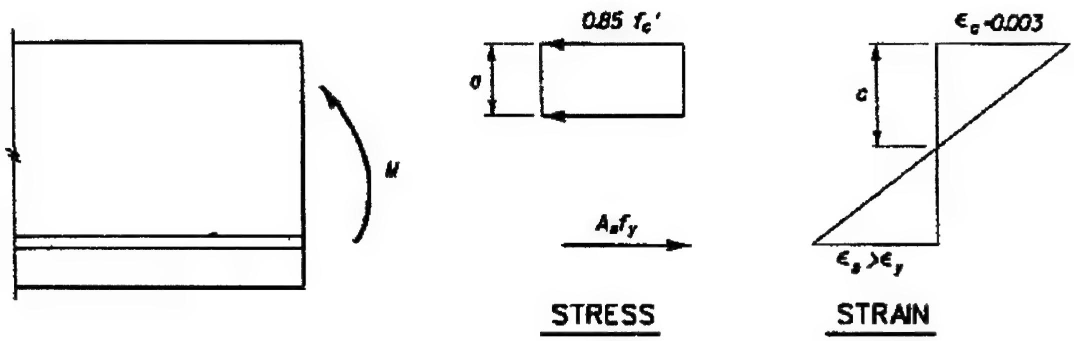


Figure E-1. Interaction diagram

E-2. Determination of Point 1, Pure Flexure



$$\phi M_n = \phi\ 0.85\ f_c'\ ab(d - a/2)$$

$$a = \frac{A_s\ f_y}{0.85\ f_c\ b} = \frac{(2.0)(60.0)}{(0.85)(3.0)(12)} = 3.922 \text{ in.}$$

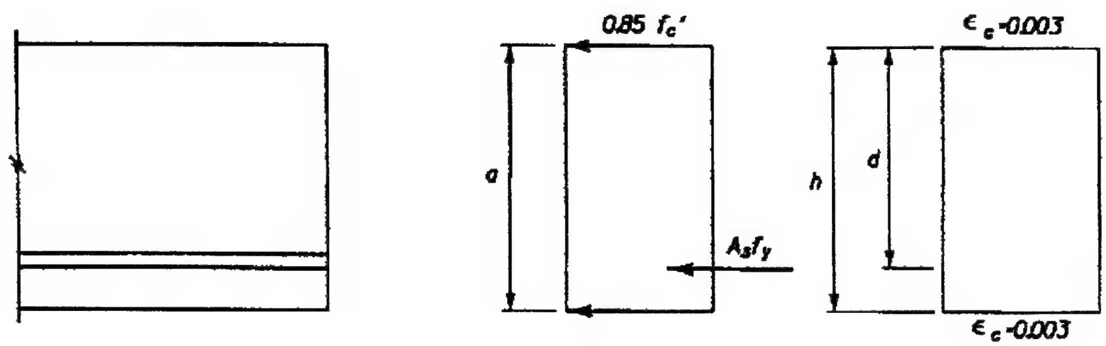
(D-5)

$$M_n = (0.85)(3.0)(3.922)(12)(22 - 1.961)$$

$$M_n = 2404.7 \text{ k-in.}$$

$$M_n = 200.4 \text{ k-ft}$$

E-3. Determination of Point 2, Maximum Axial Capacity



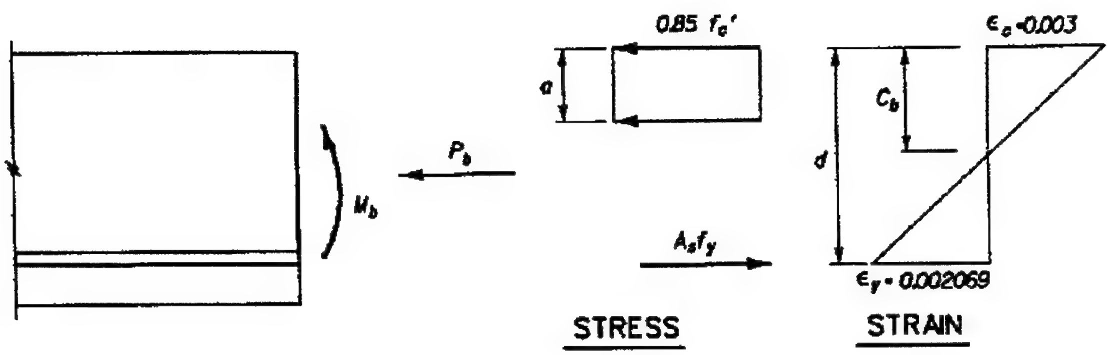
$$\phi P_{n(\max)} = \phi\ 0.80\ P_o$$

$$\phi\ P_{n(\max)} = \phi\ 0.80 \left[0.85\ f_c'\ (A_g - \rho bd) + f_y\ \rho bd\right] \tag{4-2}$$

$$P_{n(\max)} = 0.80[(0.85)(3.0)(288 - 2.0) + (60.0)(2.0)]$$

$$P_{n(\max)} = 0.80(849.3) = 679.44\ kips$$

E-4. Determination of Point 3, Balanced Point



(1) Find $k_b = \dfrac{\beta_1\ E_s\ \varepsilon_c}{E_s\varepsilon_c + f_y}$ (4-4)

$$k_b = \frac{(0.85)(29{,}000)(0.003)}{(29{,}000)(0.003) + 60} = 0.5031$$

(2) Find $\dfrac{e_b'}{d} = \dfrac{2k_u - k_u^2}{2k_u - \dfrac{pf_y}{0.425f_c'}}$ (4-3)

$$\frac{e_b'}{d} = \frac{(2)(0.5031) - (0.5031)^2}{(2)(0.5031) - \dfrac{(0.00758)(60)}{(0.425)(3.0)}} = 1.15951$$

(3) Find $\phi P_b = \phi \left[0.85\ f_c'\ k_b - \rho f_y\right] bd$

$$P_b = [(0.85)(3.0)(0.5031) - (0.00758)(60.0)](12)(22.0)$$

$$P_b = 218.62\ kips$$

(4) Find $\phi M_b = \phi \left[0.85 f_c'\ k_b - \rho f_y\right] \left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2$

$$M_b = [(0.85)(3.0)(0.5031) - (0.00758)(60)] \cdot [1.15951 - (1 - 24.0/44.0)](12)(22.0)^2 \qquad (4\text{-}6)$$

$$M_b = 3390.65 \text{ k-in.}$$

$$M_b = 282.55 \text{ k-ft}$$

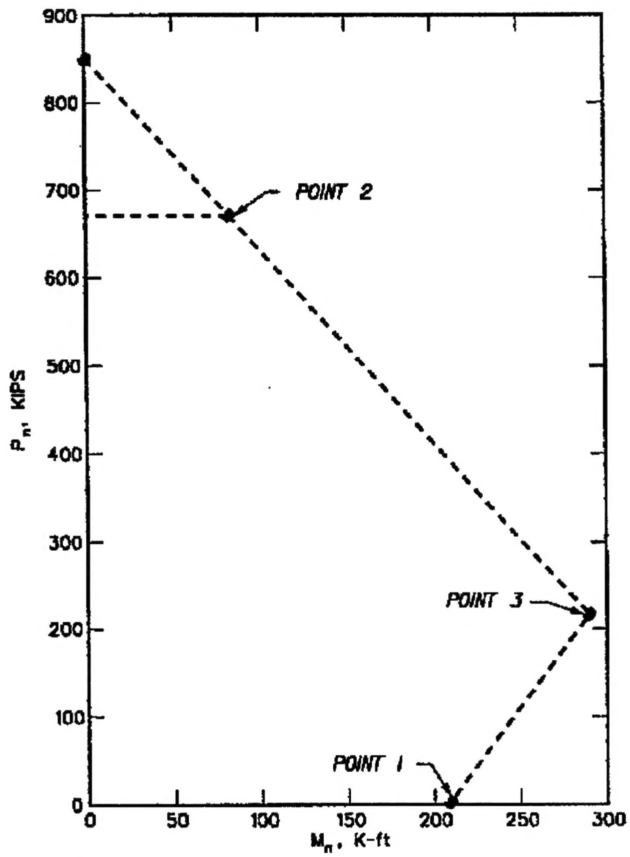


Figure E-2. Interaction diagram solution

APPENDIX F

AXIAL LOAD WITH BIAXIAL BENDING - EXAMPLE

F-1. In accordance with paragraph 4-5, design an 18- by 18-inch reinforced concrete column for the following conditions:

f'_c = 3,000 psi

f_y = 60,000 psi

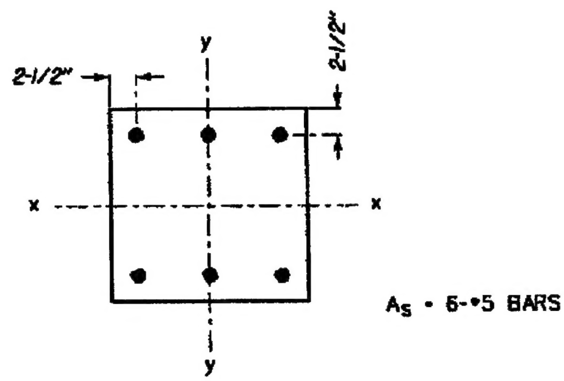
P_u = 100 kips, $P_n = P_u/0.7$ = 142.9 kips

M_{ux} = 94 ft-kips, $M_{nx} = M_{ux}/0.7$ = 134.3 ft-kips

M_{uy} = 30 ft-kips, $M_{ny} = M_{uy}/0.7$ = 42.8 ft-kips

Let concrete cover plus one-half a bar diameter equal 2.5 in.

F-2. Using uniaxial design procedures (Appendix E), select reinforcement for P_n and bending about the x-axis since $M_{nx} > M_{ny}$. The resulting cross-section is given below.



F-3. Figures F-1 and F-2 present the nominal strength interaction diagrams about x and y axes. It is seen from Figure F-2 that the member is adequate for uniaxial bending about the y-axis with P_n = 142.9 kips and M_{ny} = 42.8 ft-kips. From Figures F-1 and F-2 at P_n = 142.9 kips:

M_{ox} = 146.1 ft-kips

M_{oy} = 145.9 ft-kips

For a square column, must satisfy:

$$(M_{nx}/M_{ox})^{1.75} + (M_{ny}/M_{oy})^{1.75} \leq 1.0$$

$$(134.3/146.1)^{1.75} + (42.8/145.9)^{1.75} = 0.98 < 1.0$$

If a value greater than 1.0 is obtained, increase reinforcement and/or increase member dimensions.

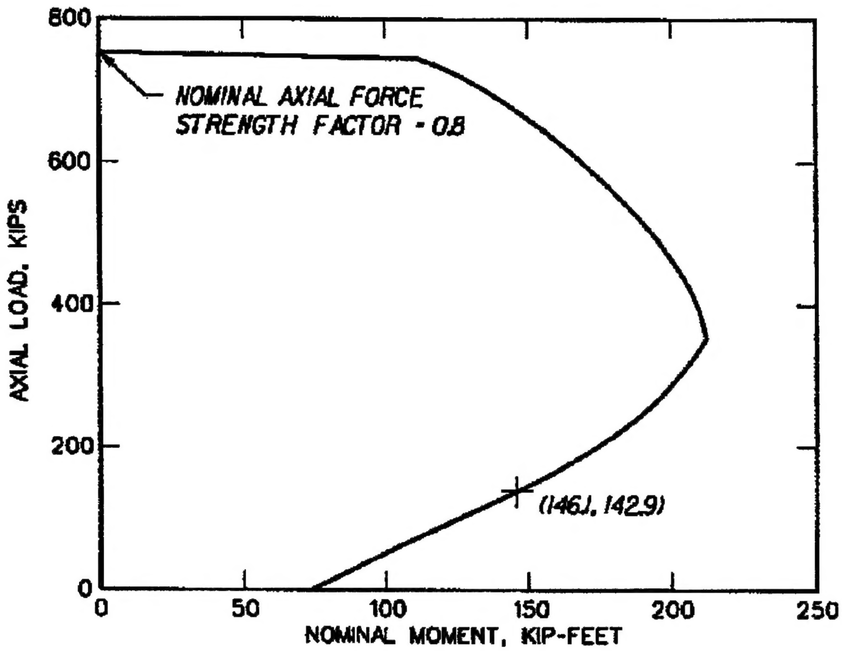


Figure F-1. Nominal strength about the X-axis

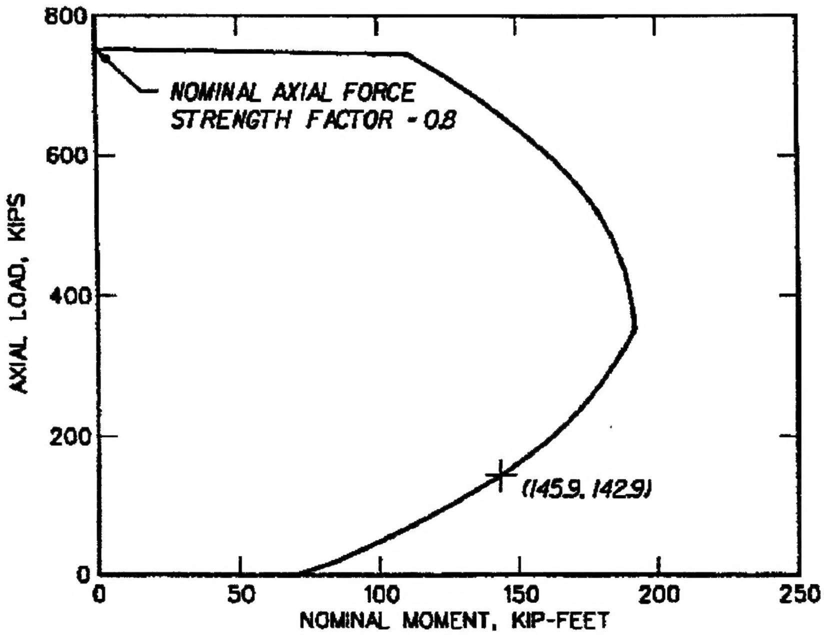


Figure F-2. Nominal strength about the Y-axis